R&D Portfolio and Market Structure*

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Abstract

This paper analyzes how firms allocate their resources when they compete for multiple patents in heterogeneous research projects simultaneously. A simple model shows that firms’ resource allocation is biased away from risky and basic research, even when imitation is not possible and firms are fully rational. Therefore a market may lack major innovations despite large aggregate research expenditure and strong patent protection. This paper also shows that as a market becomes more competitive, firms invest relatively less in basic research, but more in risky research. These results provide a novel explanation for an ambiguous empirical relationship between innovation and market concentration.

There are growing concerns about the lack of risky, long-term, and basic research\textsuperscript{1}. Common explanations include firms’ myopia, the free-rider problem, and weak intellectual property rights. These are no doubt important factors, but most of these can be addressed, at least partially, by strong patent protection. This paper, however, shows that there is another, quite basic reason due to the nature of risky and basic research, which cannot be solved by strong patent protection.

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\textsuperscript{1}“The shift away from basic research shows up repeatedly in interviews with researchers, government officials and corporate executives...many experts worry that in 20 years or so a shortfall of new technology could shackle the economy...” - New York Times, October 8, 1996
Competition, especially for patents, generates a negative externality because investment by one firm reduces other firms’ probability of winning the patent, and hence reduces their expected profits. Risky research, by definition, has a lower probability of success, and creates a smaller expected negative externality. Therefore, firms tend to relatively under-invest in risky research and over-invest in safe research.

As for basic research, by definition\(^2\), it has no commercial value by itself, and creates no negative externality. Therefore, firms tend to relatively under-invest in basic research and over-invest in applied research.

Therefore, assuming major innovations come from risky and basic research, a market may lack innovations even with a large aggregate R&D expenditure. Then, economic analysis or policy based on aggregate R&D measures (e.g. number of patents, R&D expenditures) can be misleading.

As far as I know, this is the first paper that shows firms would under-invest in basic and risky research, even when all research projects are protected by patents and when firms are fully rational. That is, the free-rider problem or myopic firms are not fully responsible for the lack of basic or risky research. Then, even the strongest patent protection will not be sufficient to promote risky and basic research. However, most policy discussions have focused on strengthening intellectual property rights. In fact, Bessen and Maskin (2006) and Kwon (2008) show that strong patent protection for complementary projects such as basic and applied research may reduce firms’ research incentives. Therefore, alternative policies such as subsidies or financing specifically targeted at risky and basic research may be necessary.

Some also argue that competition exacerbates the lack of basic research, because firms become more short-sighted or because each innovation has more imitators. Yet, despite growing global competition, there exist few theoretical studies on the role of competition in basic research\(^3\). This

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\(^2\)The National Scientific Foundation (1959) defines basic research as “original investigation for the advancement of scientific knowledge... which do(es) not have immediate commercial objectives.”

\(^3\)Kato (2005) studies the role of competition and the allocation of R&D expenditure between product innovation and process innovation, but not between basic and applied research. Pioneering empirical studies by Mansfield (1981) and Link (1982) analyze the composition of R&D and market structure using cross-industry data, but do
paper shows that competition does reduce relative spending in basic research. It is worth stressing that this result holds even when imitation is not possible and when firms are rational. Therefore, again, strengthening patent protection would not mitigate the effect of competition.

Interestingly, however, competition increases relative spending in risky, long-term research. Therefore, when innovation mostly relies on basic research, it may be slowed down by competition. However, when innovation mostly relies on risky research, it may be speeded up by competition. These results provide a novel explanation for the ambiguous empirical relationship between market structure and innovation. For example, using aggregate measures only, Levin et al. (1985, 1987) and Cohen et al. (1987) found no significant relationship between market concentration and innovation. This paper suggests that distinguishing different types of innovations can be important in uncovering the true relationship between innovation and market structure.

There is a series of theoretical studies on R&D portfolio and its market bias (e.g. Bhattacharya and Mookherjee 1986, Klette and de Meza 1986, Dasgupta and Maskin 1987, and Cabral 1994). However, they exclusively focus on the resource allocation between risky and safe research, and do not study the allocation between basic and applied research or its relationship with market structure.

1 Model

There are $n$ symmetric firms and two different projects ($A$ and $B$). Following previous studies on R&D portfolio (e.g. Dasgupta and Maskin 1987 and Cabral 1994), I normalize each firm’s total not control for unobserved industry or firm-specific effects.

4 For a detailed survey, see Cohen and Levin (1986).

5 There is also a series of studies on sequential innovations (e.g. Scotchmer 1991, O’Donoghue et. al. 1998, and Bessen and Maskin 2006) where intermediate innovations can be interpreted as basic research. However, firms in these models do not invest in more than one project at any given time. Thus, these models do not capture the competition for R&D portfolios. Moreover, none of these studies analyze the market bias in relative resource allocation.

6 Other studies also analyzed how a market structure affects firms incentives for drastic vs. non-drastic innovations (e.g. Reinganum 1983) or for process vs. product innovations (e.g. Cohen and Klepper 1996, Kato 2005). However, none studies the market bias in resource allocation.
investment to one, and denote firm \(i\)'s investment in project \(A\) by \(x_i\) where \(0 \leq x_i \leq 1\). The projects can either succeed or fail. The probability of success of firm \(i\) in project \(A\) is \(p_i = p(x_i)\). Similarly, for project \(B\), the probability of success is \(q_i = q(1 - x_i)\). Both \(p(.)\) and \(q(.)\) are increasing and strictly concave.

Firms compete for a winner-take-all patent in each project. If multiple firms succeed in the same project, each wins the patent with the same probability. Then, in a symmetric equilibrium, firm \(i\)'s probability of winning a patent for project \(A\) is:

\[
p_i (n-1)C_0 p^0 (1-p)^{n-1} + \frac{n-1} {2} p_1 (1-p)^{n-2} + \ldots + \frac{n-1} {n} p_n (1-p)^0 \]

\[
= \frac{p_i} {np} \left( \sum_{l=0}^{n} \binom{n}{l} p^l (1-p)^{n-l} - (1-p)^n \right) \]

\[
= \frac{p_i} {np} (1 - (1-p)^n) \] (1)

For convenience, firm \(i\)'s probability of success will be noted with the subscript \(i\). Given the symmetry, all the other firms’ probabilities of success will be written without subscripts.

I consider two different cases of research heterogeneity. In the first case, project \(B\) is riskier than project \(A\). More specifically, suppose that the value of project \(j\) is \(V_j\). Then, \(q(.) = \frac{p(.)}{\theta}\) and \(V_B = \theta V_A\) where \(\theta > 1\). Therefore, the two projects have the same expected value, but project \(B\) has a lower probability of success.

In the second case, the two projects are complementary but project \(B\) has no value by itself. More specifically, if firm \(i\) wins patents for both project \(A\) and \(B\), it earns \(V_{AB}\), where \(V_{AB} > V_A + V_B\). Also, \(V_B = 0\). I interpret project \(B\) as basic research, and project \(A\) as applied research. Even though it is not the intention of this paper to capture the sequential aspect of basic and applied research, simultaneous investment in basic and applied research is becoming more common.

\(^{7}\) For simplicity, assume that firms can appropriate all the social value of innovations. Even if firms appropriate only a fixed proportion of the social value, the results of this paper do not change.

\(^{8}\) See Scotchmer (1991), O’Donoghue et. al. (1998), and Bessen and Maskin (2006) for sequential innovations. However, they do not study the bias in relative resource allocation between basic and applied research.

\(^{9}\) Today research and development occurs simultaneously. There’s no opportunity to do research first and develop the technology gradually. - from "Where the Fantastic Meets the Future" Business Week, August 25, 2004
2 Market Bias in R&D Portfolio

This section analyzes whether firms tend to relatively over- or under-invest in risky or basic research, compared with the social optimum.

2.1 Risky vs. Safe Research

Suppose project B is riskier than project A as described above. From the social point of view, it suffices if at least one firm succeeds in each project. Therefore, assuming a symmetric optimum, the expected social welfare, denoted by $V_S$, is as follows:

$$V_S(x_1, \ldots, x_i, \ldots, x_n) = (1 - (1 - p_i)(1 - p)^{n-1})V_A + (1 - (1 - q_i)(1 - q)^{n-1})V_B$$  \hspace{1cm} (2)

Let us denote the symmetric social optimum by $x^*_S$.

However, firm $i$ can make profits only if it wins the patent. Therefore, firm $i$’s expected profit is as follows:

$$\pi_i(x_1, \ldots, x_i, \ldots, x_n) = \frac{p_i}{np} (1 - (1 - p)^n)V_A + \frac{q_i}{nq} (1 - (1 - q)^n)V_B$$  \hspace{1cm} (3)

Also, denote the symmetric market equilibrium by $x^*_M$.

Proposition 1 Compared with the social optimum, firms invest relatively more in safe projects, or relatively less in risky projects. That is, $x^*_M > x^*_S$.

Proof. See appendix. ■

Therefore, firms tend to relatively under-invest in risky (or long-term\(^{10}\)) projects. Intuitively, in a patent race, R&D investment creates a negative externality since it reduces the rival’s probability of winning a patent, and hence the expected profit. Note that this externality arises when

\(^{10}\)A risky project can be also interpreted as a long-term project. For example, consider the Poisson process where the probability of the random success date in a project, $\tau(x)$, is less than $t$, is given by $\Pr(\tau(x) < t) = 1 - e^{-h(x)t}$. Then the expected success date is $\frac{1}{h(x)}$ and the expected value of the R&D investment is $\int_0^\infty e^{-rt}h(x)e^{-h(x)t}dt = V \frac{h(x)}{r + h(x)}$. If we define $\frac{h(x)}{r + h(x)}$ as $p(x)$, then the long-term project has lower $p(.)$.\]
both firms succeed in the same project. Even though both safe and risky projects have the same expected values, the probability of joint success in safe projects is much larger than that in risky projects. Thus, safe projects have a larger negative externality. In other words, compared with the social optimum, there will be relative over-investment in safe projects or under-investment in risky projects.¹¹

Note that this result is the opposite of the results from Klette and de Meza (1986), Bhattacharya and Mookerjee (1986), and Dasgupta and Maskin (1987). These studies have shown that firms tend to over-invest in risky research strategies. The main difference is that these studies have considered firms’ selection among different research strategies for one given project or patent, while I am considering resource allocation among different research projects for multiple patents. Cabral (1994) shows similar results to proposition 1, but does not consider complementary projects or market structure as analyzed in the next sections.

2.2 Basic vs. Applied Research

Now suppose that project A is applied research and that project B is basic research. Thus, if at least one firm succeeds in both basic research and applied research, society can enjoy the additional complementary value \((V_{AB} - V_A)\). For simplicity, I assume that new innovation cannot be transferred across firms¹². Then, the expected social welfare is as follows:

\[
V_S(x_1, \ldots, x_i, \ldots, x_n) = (1 - (1 - p_i q_i)(1 - pq)^{n-1})(V_{AB} - V_A) + (1 - (1 - p_i)(1 - p)^{n-1})V_A
\]

Recall that project B has no value by itself.¹³ With a slight abuse of notation, let us denote the symmetric social optimum by \(x_S^*\).

¹¹ Alternatively speaking, from a social point of view, if firm \(i\) succeeds in a project, firm \(j\)’s success in the same project has no additional social value. However, from firm \(j\)’s point of view, its success still has a value as long as it has a chance to win the patent. Therefore, there exists a negative externality. Because the probability of joint success is lower for risky projects, firms tend to relatively over-invest in safe projects.

¹² Allowing transfer of technology and licensing does not change the qualitative results of this paper, as long as firms with a patent for applied innovation have enough bargaining power.

¹³ Even if there are multiple applied research projects, as long as they are independent in probability of success, the results of this paper do not change.
Firms, however, must win patents for both project A and B to profit from the complementarity between the two projects. Thus, the expected profit in a symmetric equilibrium is as follows:

$$\pi_i(x_1, \ldots, x_i, \ldots, x_n) = \frac{p_i}{np}(1 - (1 - p)^n)q_i(1 - (1 - q)^n)(V_{AB} - V_A) + \frac{p_i}{np}(1 - (1 - p)^n)V_A$$

(5)

Again, denote the symmetric market equilibrium by $x^*_M$.

**Proposition 2** Compared with the social optimum, firms invest relatively more in applied projects, or relatively less in basic projects. That is, $x^*_M > x^*_S$.

**Proof.** See appendix. ■

Therefore, firms do under-invest in basic research, as is commonly feared. It is worth stressing that the under-investment in basic research arises not because of the ‘free-rider’ problem, as basic research is fully protected by patents in this model. It is simply because basic research investment in one firm has no negative externality on basic research investment in another firm as it has no commercial value by itself\(^{14}\). Therefore, while many argue for stronger patents to promote basic research, this paper suggests that strong patent protection would not be enough. In fact, in a related model, Kwon (2008) shows that strong patent protection can reduce the absolute amount of research investment.

Most previous studies and policy discussions have largely relied on aggregate R&D measures. However, assuming that major innovations come from risky and basic research, Proposition 1 and 2 imply that the focus on such aggregate measures can be misleading. In particular, if firms do not invest in risky and basic research enough, as proposition 1 and 2 show, a market may not be innovative despite large aggregate R&D expenditures.

A caveat is, though, that this paper only focuses on relative investment. Therefore, if firms invest too much overall, relative under-investment in risky and basic research does not necessarily

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\(^{14}\)Basic research still has some negative externality because holding the patent for basic research makes the other firm’s patent for applied research less useful, often called the ‘hold-up’ effect. (see, e.g., Shapiro 2001, Bessen and Maskin 2006, and Kwon 2008 for more details). However, the hold-up effects apply to both basic and applied research, and they cancel out each other in this model.
mean under-investment in absolute amount. However, since firms typically cannot fully appropriate the social value of innovations, they tend to under-invest in absolute amount. Then, relative under-investment in basic and risky research in this paper should imply under-investment in this research by absolute amount too.

3 R&D Portfolio and Market Structure

The previous section shows that firms do relatively under-invest in basic, risky and long-term research, as commonly feared. Some also argue that under-investment in these types of research has become more serious due to increasing global competition, possibly because firms become more short-sighted or because there are more firms trying to imitate innovations. However, most previous studies have focused on competition and aggregate R&D spending.

To analyze how the market equilibrium changes with competition in our model, it is sufficient to check how the number of firms will change the relative marginal returns of the two projects, i.e., the sign of $\frac{\partial^2 \pi_i}{\partial n \partial x_i}(x^*_M)$. The following proposition shows that competition affects relative investment in basic research and risky research in opposite way.

**Proposition 3** As the number of firms, $n$, increases, the relative expenditure in risky projects increases, but the relative expenditure in basic projects decreases.

**Proof.** See appendix. ■

Therefore, competition does decrease relative investment in basic research, even though firms are fully rational and all research outcomes are fully protected by patents in our model. Even though some argue for stronger patent protection in response to growing competition, this result implies that stronger patent protection would not necessarily reduce the effect of competition.

Interestingly, however, competition increases relative investment in risky, long-term research. Contrary to the idea that competition will make firms myopic, this result suggests that firms will become relatively more far-sighted in terms of research portfolios.

Intuitively, since the safe project has a larger probability of success, the probability of losing the patent race in a safe project increases rapidly as the number of competitors increases. On the
other hand, since the risky project has a small probability of success, the number of competitors has a smaller effect on the probability of losing the patent. Therefore, as the number of firms increases, the firms will invest relatively more in risky projects.

A basic project can realize commercial value only together with an applied project. Since the probability of winning the patent decreases as the number of firms increases, firms will invest relatively more in applied projects as the number of firms increases.

Assuming that major innovations come from risky and basic research, these results explain why the empirical relationship between innovation and market competition has been ambiguous. For example, earlier studies found a positive relationship between market concentration and innovation, but more recent studies (e.g. Levin et al. 1985, 1987, Cohen et al. 1987) found no significant relationship or found non-linear relationship (see Cohen and Levin 1986 for a detailed survey). Proposition 3 shows that as a market becomes more concentrated (or as $n$ decreases), innovations based on risky projects will decrease, but innovations based on basic research will increase. Therefore, if a study focuses on aggregate measures of innovation, the relationship between innovation and market structure can be ambiguous.

4 Discussion

From the policy perspective, this paper provides at least three implications. First, most current policy discussions are based on aggregate R&D measures (such as number of patents, R&D expenditure). For example, in the 2007 OECD Science, Technology, Industry Scoreboard\textsuperscript{15}, most statistics focus on aggregate measures without distinguishing basic and applied research or risky and safe research. However, this paper shows that focusing on aggregate measures can be misleading. Since firms under-invest in risky and basic research, if a government simply tries to increase the aggregate R&D spending, a market may not be innovative despite a large R&D expenditure.

Second, stronger patent protection or intellectual property rights has been one of the typical policy responses to the lack of basic or risky research. For example, in the US, patent protection

\textsuperscript{15}OECD Science, Technology, Industry Scoreboard 2007\textsuperscript{a}, 2007, 20 February 2008

< http://caliban.sourceoecd.org/vl=4626228/cl=24/nw=1/rpsv/sti2007/>
has been strengthened over the years, as highlighted by the 1982 formation of a centralized appellate court, the Court of Appeals for the Federal Circuit. This paper shows, however, that stronger patent protection for risky and basic research may not be sufficient, and may even be harmful. Under-investment in basic and risky research arises due to the very nature of these types of research, not necessarily because of weak patent protection. In fact, Bessen and Maskin (2006) and Kwon (2008) show that strong patent protection can even decrease firms’ research incentives. Therefore, alternative policies, such as government subsidies, or tax relief targeted specifically at basic and risky research, or creating stronger incentives for government funded institutions to focus on basic and risky research may be necessary.

Third, the policy priority toward risky or basic research may depend on the market concentration. Proposition 3 implies that in a concentrated market, firms would invest relatively more in basic research and less in risky research. Then, policies supporting risky research would be more desirable. However, in a competitive market, firms invest relatively less in basic research and more in risky research. Then, policies supporting basic research should be a priority. In particular, as an economy becomes more globalized, the increasing competition should require more policies focusing on basic research, possibly by government funded research institutions such as universities.

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References


Appendix

Proof of Proposition 1

Claim 1  \( p(x_S^*) < p(1 - x_S^*), \) \( p'(x_S^*) > p'(1 - x_S^*), \) and \( p(x_S^*) > \frac{p(1 - x_S^*)}{\theta}. \)

The first order condition for social welfare maximization is as follows:

\[
\frac{\partial V_S}{\partial x_i}(x_S^*) = p'(x_S^*)(1 - p(x_S^*))^{n-1} V_A - \frac{p'(1 - x_S^*)}{\theta} (1 - \frac{p(1 - x_S^*)}{\theta})^{n-1} \theta V_A = 0 \tag{A.1}
\]

Recall that \( V_B = \theta V_A, \) \( q_i = \frac{p(1-x_i)}{\theta}, \) and \( \theta > 1. \)

Suppose that \( x_S^* = \frac{1}{2}. \)

\[
\frac{\partial V_S}{\partial x_i}(\frac{1}{2}) = p'(\frac{1}{2})(1 - p(\frac{1}{2}))^{n-1} V_A - \frac{p'(\frac{1}{2})}{\theta} (1 - \frac{p(\frac{1}{2})}{\theta})^{n-1} \theta V_A \tag{A.2}
\]

The inequality is due to \( \theta > 1. \) Since \( \frac{\partial V_S}{\partial x_i}(x_S^*) \) is decreasing in \( x_S^*, \) \( x_S^* \) must be less than \( \frac{1}{2}. \) Therefore, \( p(x_S^*) < p(1 - x_S^*) \) and \( p'(x_S^*) > p'(1 - x_S^*) \) since \( p'' < 0. \) Then from (A.1), \( p(x_S^*) > \frac{p(1-x_S^*)}{\theta}. \)

Claim 2  \( \frac{\partial \pi_i}{\partial x_i}(x_S^*) = \frac{p(x_S^*)}{n} V_A - \frac{p'(1-x_S^*)}{n} V_A. \)

The derivative of a firm \( i \)'s profit function at the symmetric social optimum is as follows:

\[
\frac{\partial \pi_i}{\partial x_i}(x_S^*) = \frac{p_i}{np}(1 - (1-p)^n) V_A - \frac{q_i'}{nq} (1 - (1-q)^n) V_B \tag{A.3}
\]

\[
= \frac{p_i}{np} (1 - (1-p)(1-p)^{n-1}) V_A - \frac{q_i'}{nq} (1 - (1-q)(1-q)^{n-1}) V_B \\
= \frac{p_i}{np} (1 - (1-p)^{n-1} + p(1-p)^{n-1}) V_A - \frac{q_i'}{nq} (1 - (1-q)^{n-1} + q(1-q)^{n-1}) V_B \\
= \left[ \frac{p_i}{np} (1 - (1-p)^{n-1}) V_A - \frac{q_i'}{nq} (1 - (1-q)^{n-1}) V_B \right] \\
+ \left[ \frac{p_i}{n} (1 - p)^{n-1} V_A - \frac{q_i'}{n} (1 - q)^{n-1} V_B \right] \\
= \frac{p_i}{np} (1 - (1-p)^{n-1}) V_A - \frac{q_i'}{nq} (1 - (1-q)^{n-1}) V_B 
\]
The last equality is due to \[ \frac{\partial }{\partial x_i} (1 - p)^{n-1} V_A - \frac{q_i}{n} (1 - q)^{n-1} V_B \] = 0 at \( x = x^*_i \) from (A.1). By the same logic,

\[
\frac{\partial \pi_i}{\partial x_i} (x^*_S) = \frac{p'_i}{np} (1 - (1 - p)^{n-1}) V_A - \frac{q'_i}{nq} (1 - (1 - q)^{n-1}) V_B \\
= \frac{p'_i}{np} (1 - (1 - p)^{n-2}) V_A - \frac{q'_i}{nq} (1 - (1 - q)^{n-2}) V_B \\
= ... \\
= \frac{p'_i}{np} (1 - (1 - p)) V_A - \frac{q'_i}{nq} (1 - (1 - q)) V_B \\
= \frac{p'_i}{n} V_A - \frac{q'_i}{n} V_B = \frac{p(x^*_S)^{'}}{n} V_A - \frac{p'(1 - x^*_S)}{n} V_A \\
\] (A.4)

Claim 3 \( x^*_M > x^*_S \).

Since \( \frac{\partial \pi_i}{\partial x_i} (x^*_M) \) is decreasing in \( x^*_M \), it is sufficient to show \( \frac{\partial \pi_i}{\partial x_i} (x^*_S) > 0 \). From claim 1, \( p'(x^*_S) > p'(1 - x^*_S) \). Then, from claim 2, \( \frac{\partial \pi_i}{\partial x_i} (x^*_S) > 0 \).

Proof of Proposition 2

Claim 1 \( p'(x^*_S) q (1 - x^*_S) - p(x^*_S) q' (1 - x^*_S) < 0 \).

The first order condition for social welfare maximization is as follows:

\[
\frac{\partial V_S}{\partial x_i}(x^*_S) = (p' q - pq')(1 - pq)^{n-1} (V_{AB} - V_A) + p'(1 - p)^{n-1} V_A = 0 \\
\] (A.5)

Since \( V_{AB} - V_A > 0 \), it must be that \( (p' q - pq') < 0 \).

Claim 2 \( \frac{\partial \pi_i}{\partial x_i} (x^*_S) > 0 \).

The derivative of firm \( i \)'s profit function at the symmetric social optimum is as follows:

\[
\frac{\partial \pi_i}{\partial x_i} (x^*_S) = (p' q - pq') \frac{1}{n^2 pq} (1 - (1 - p)^n)(1 - (1 - q)^n)(V_{AB} - V_A) + p' \frac{1}{np} (1 - (1 - p)^n) V_A \\
\] (A.6)

Since \( p' q - pq' < 0 \) at \( x^*_S \), in order to prove \( \frac{\partial \pi_i}{\partial x_i} (x^*_S) > 0 \), it is sufficient to show

\[
\frac{-(p' q - pq') \frac{1}{n^2 pq} (1 - (1 - p)^n)(1 - (1 - q)^n)(V_{AB} - V_A)}{p' \frac{1}{np} (1 - (1 - p)^n) V_A} < 1
\]

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From (A.5),
\[ \frac{V_{AB} - V_A}{V_A} = \frac{p'(1-p)^{n-1}}{-(pq' - pq)(1-pq)^{n-1}} \]  
(A.7)

Substituting (A.7) into (A.6) yields
\[ \frac{-(pq' - pq)(1 - (1 - p)^n)(1 - (1 - q)^n)}{p'\frac{1}{np}(1 - (1 - p)^n)} \frac{p'(1-p)^{n-1}}{-(pq' - pq)(1-pq)^{n-1}} \]
\[ = \frac{\frac{1}{nq}(1 - (1 - q)^n)(1 - p)^{n-1}}{(1-pq)^{n-1}} < 1 \]

The inequality follows from \( \frac{1}{nq}(1 - (1 - q)^n) < 1 \) and \((1 - p)^{n-1} < (1 - pq)^{n-1} \).

**Claim 3** \( x_M^* > x_S^* \).

Since \( \frac{\partial \pi}{\partial x_i}(x_M) \) is decreasing in \( x_M \) and \( \frac{\partial \pi}{\partial x_i}(x_S^*) > 0 \) from claim 2, it must be that \( x_M^* > x_S^* \).

**Proof of Proposition 3**

For risky and safe projects, from equation (3) in the text, the first order condition for a market equilibrium is as follows:
\[ \frac{\partial \pi_i}{\partial x_i}(x_M^*) = V_A\frac{p'}{np}(1 - (1 - p)^n) - V_B\frac{q'}{nq}(1 - (1 - q)^n) = 0 \]  
(A.8)

Since \( \frac{\partial \pi_i}{\partial x_i} \) is decreasing, it is sufficient to show that \( \frac{\partial^2 \pi_i}{\partial n\partial x_i}(x_M^*) < 0 \). That is, if \( n \) decreases the marginal profit from a safe project, the relative investment in a risky project will increase.
\[ \frac{\partial^2 \pi_i}{\partial n\partial x_i}(x_M^*) = -\frac{1}{n}[V_A\frac{1}{np}(1 - (1 - p)^n)p'_i - V_B\frac{1}{nq}(1 - (1 - q)^n)q'_i] \]
\[ -[V_A\frac{p'}{np}(1 - p)^n\log(1 - p) - V_B\frac{q'}{nq}(1 - q)^n\log(1 - q)] \]
\[ = -[V_A\frac{p'}{np}(1 - p)^n\log(1 - p) - V_B\frac{q'}{nq}(1 - q)^n\log(1 - q)] \]  
(A.9)

The second equality is from (A.8).
To show \( \frac{\partial^2 \pi_i}{\partial q \partial x_i}(x_M^*) < 0 \), it is sufficient to show:

\[
\begin{align*}
\frac{-V_A \frac{q'}{np}(1-p)^n \log(1-p) - V_B \frac{q'}{nq}(1-q)^n \log(1-q)}{1} & < 1 & \iff & \frac{-(1-p)^n \log(1-p)^n}{-(1-q)^n \log(1-q)^n} < \frac{V_B q'}{V_A p'} \quad \text{(A.10)}
\end{align*}
\]

The second equivalence is from (A.8). Recall that \( q(x) = p(1-x)/\theta \) and \( V_B = \theta V_A \) where \( \theta > 1 \). Therefore, from (A.8), \( p(x_M^*) > q(x_M^*) \). Then, it is sufficient to show that \( \frac{(1-p)^n \log(1-p)^n}{(1-q)^n \log(1-q)^n} \) is increasing in \( p \), or \( G(a) \equiv \frac{a^n \log(a^n)}{1-a^n} \) is decreasing in \( a \) for \( 0 \leq a \leq 1 \). In fact, \( \text{sign}(G'(a)) = \text{sign}(\log(a+1)(1-a) + a \log a) = \text{sign}(1-a + \log a) < 0 \), since \( \log a - a \) has its maximum -1 at \( a=1 \).

For basic and applied projects, from equation (6) in the text, the first order condition for a market equilibrium is as follows:

\[
\frac{\partial \pi_i}{\partial x_i}(x_M^*) = (p'q - pq') \frac{1}{np}(1-(1-p)^n) + \frac{1}{nq}(1-(1-q)^n)(V_{AB} - V_A) + p' \frac{1}{np}(1-(1-p)^n)V_A
\]

\[
= (p'q - pq') \frac{1}{nq}(1-(1-q)^n)(V_{AB} - V_A) + p'V_A = 0
\]  

(A.11)

Since \( \frac{\partial \pi_i}{\partial x_i} \) is decreasing, it is sufficient to show that \( \frac{\partial^2 \pi_i}{\partial n \partial x_i}(x_M^*) > 0 \). That is, if \( n \) increases the marginal profit from an applied project, the relative investment in a basic project will decrease.

\[
\begin{align*}
\frac{\partial^2 \pi_i}{\partial n \partial x_i}(x_M^*) & = -(p'q - pq') \frac{1}{n^2q}(1-(1-q)^n)(V_{AB} - V_A) + (p'q - pq') \frac{1}{nq}(1-q)^n \log(1-q)(V_{AB} - V_A) \\
& = -(p'q - pq')(V_{AB} - V_A)(1-(1-q)^n + n(1-q)^n \log(1-q)) \\
& = -(p'q - pq') (V_{AB} - V_A)(1-(1-q)^n + (1-q)^n \log(1-q)^n)
\end{align*}
\]  

(6)

Since \( p'q - pq' < 0 \) at \( x_M^* \) from (A.11), it is sufficient to show that \( 1-(1-q)^n + (1-q)^n \log(1-q)^n > 0 \) or \( 1-a + a \log(a) > 0 \) for \( 0 < a < 1 \). In fact, \( 1-a + a \log(a) \) has its minimum 0 at \( a = 1 \), where \( 0 < a = (1-q)^n < 1 \).